Entanglement Islands in Braneworlds

Based on: arxiv: 2006.02438, JHEP 09 (2020) 121;
arxiv: 2012.04671;
arxiv: 2103.07477;
arxiv: 2103.xxxxx.

Hao Geng

(University of Washington)

Plan:

- Brief Review of Karch-Randall (KR) Brane-world (Karch and Randall 01)
- An Information Paradox (Hartman, Maldacena 13; Mathur 14; Almheri, Mahajan and Maldacena 19)
- Quantum Extremal Surface and Double Holography (Almheri, Mahajan and Santos 19)
- Analytic Resolution of the Paradox in KR (HG and Karch 20)
- **Massive Graviton and Islands**
- An Attempt to Massless: Information Transfer with a Gravitating Bath

(HG, Karch, Perez, Raju, Randall, Riojas and Shashi 20)

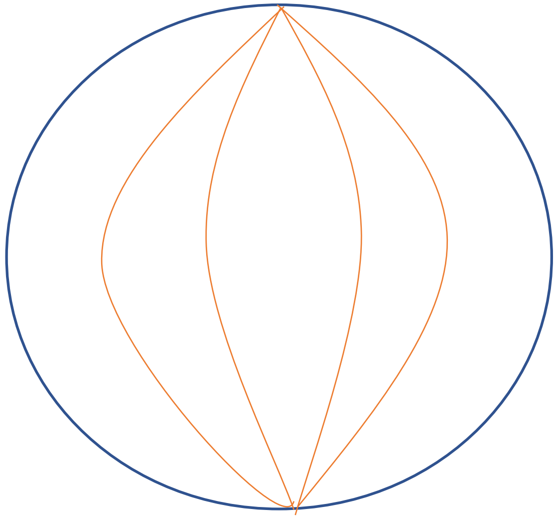
- A Parallel Story in de Sitter (HG, Nomura and Sun to appear)

KR Brane-world and Its Three Descriptions

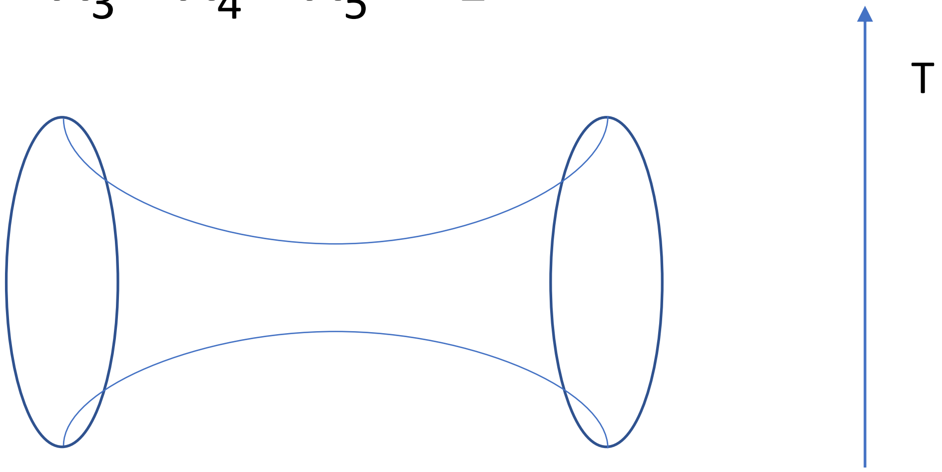
(Karch and Randall 01, a recent generalization: Karch and Randall 20)

- Basics of AdS:

1) AdS_5 space: $-X_0^2-X_1^2+X_2^2+X_3^2+X_4^2+X_5^2=-L^2$



AdS_5 constant time slice in global coordinate

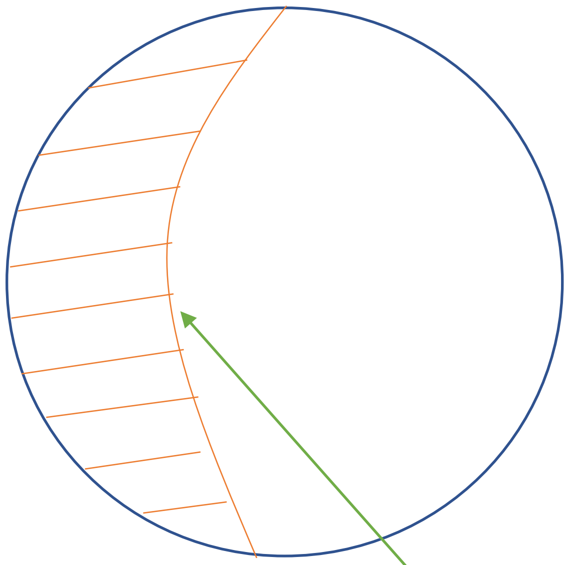


AdS_5 in embedding space

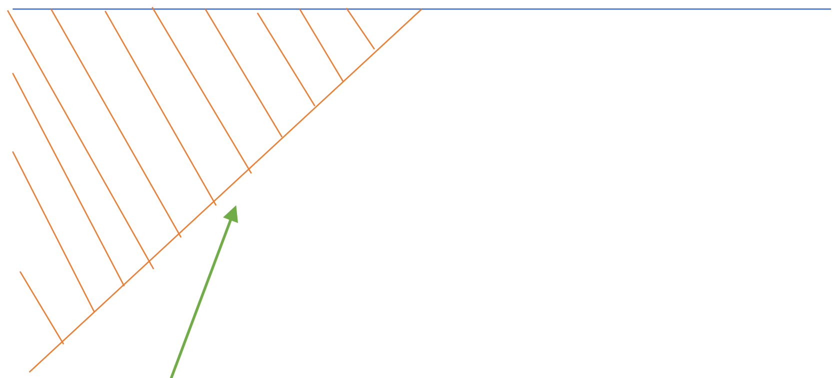
2) AdS_5/AdS_4 : $-X_0^2-X_1^2+X_2^2+X_3^2+X_4^2=-L^2\cosh^2(r)$

$$X_5^2=L^2\sinh^2(r)$$

- Karch-Randall Brane:



Constant time slice in global patch



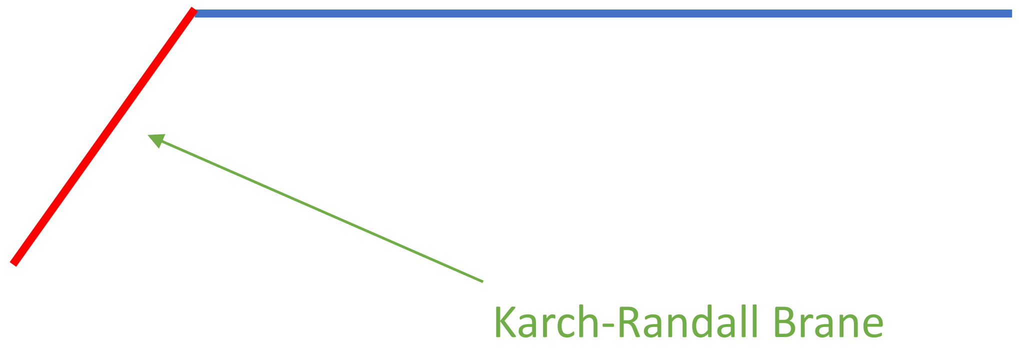
Constant time slice in Poincare patch

Karch-Randall Brane AdS_4

$$S = \frac{1}{16\pi G}\int d^5x\,\sqrt{-g}(R-2\Lambda) - \frac{1}{8\pi G}\int_{brane} d^4x\,\sqrt{-h}\,\,(K+T)$$

- Three Descriptions of KR Brane-world:

(1) 5d Einstein Gravity in AdS_5 with a Karch-Randall Brane:



holography 1

(2) 4d (holographic) CFT Coupled to a 4d Gravitating (holographic) CFT via Transparent Boundary Condition:

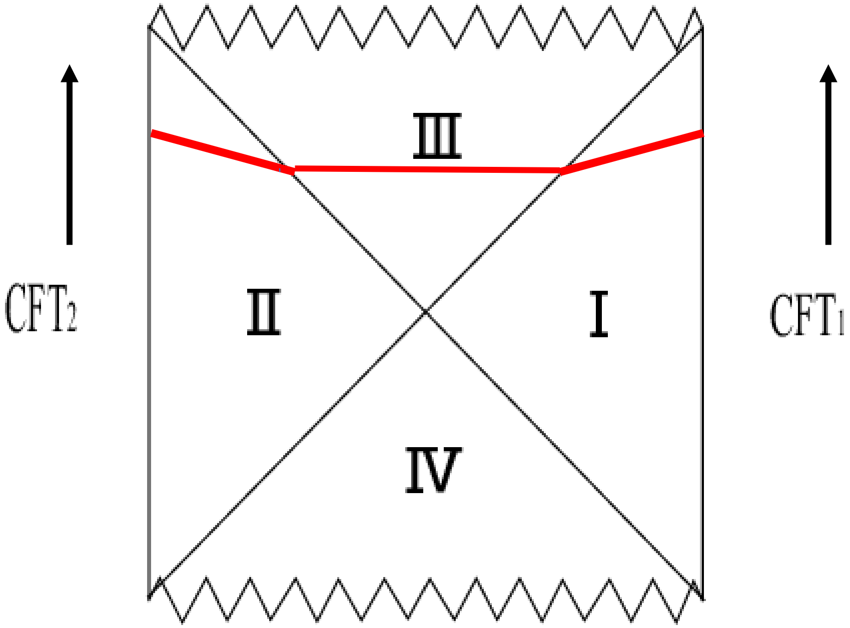
holography 2

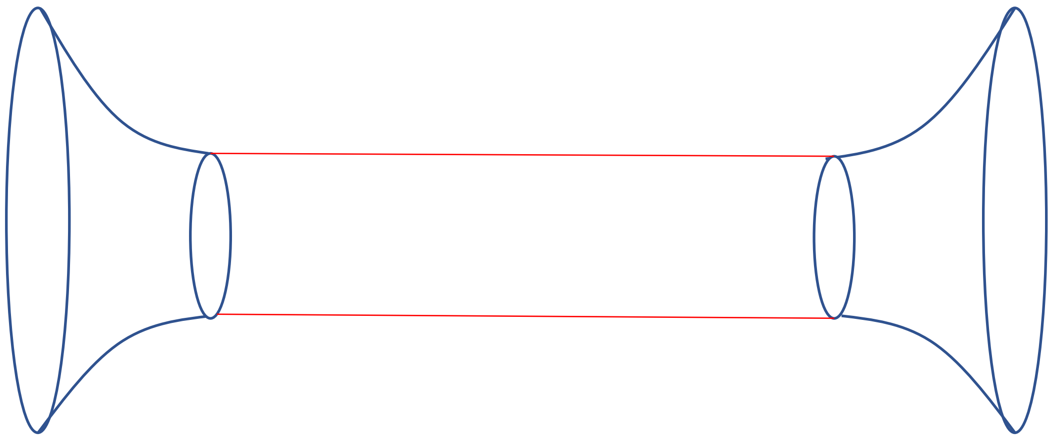
(3) 4d Boundary (holographic) Conformal Field Theory (BCFT):

An Information Paradox

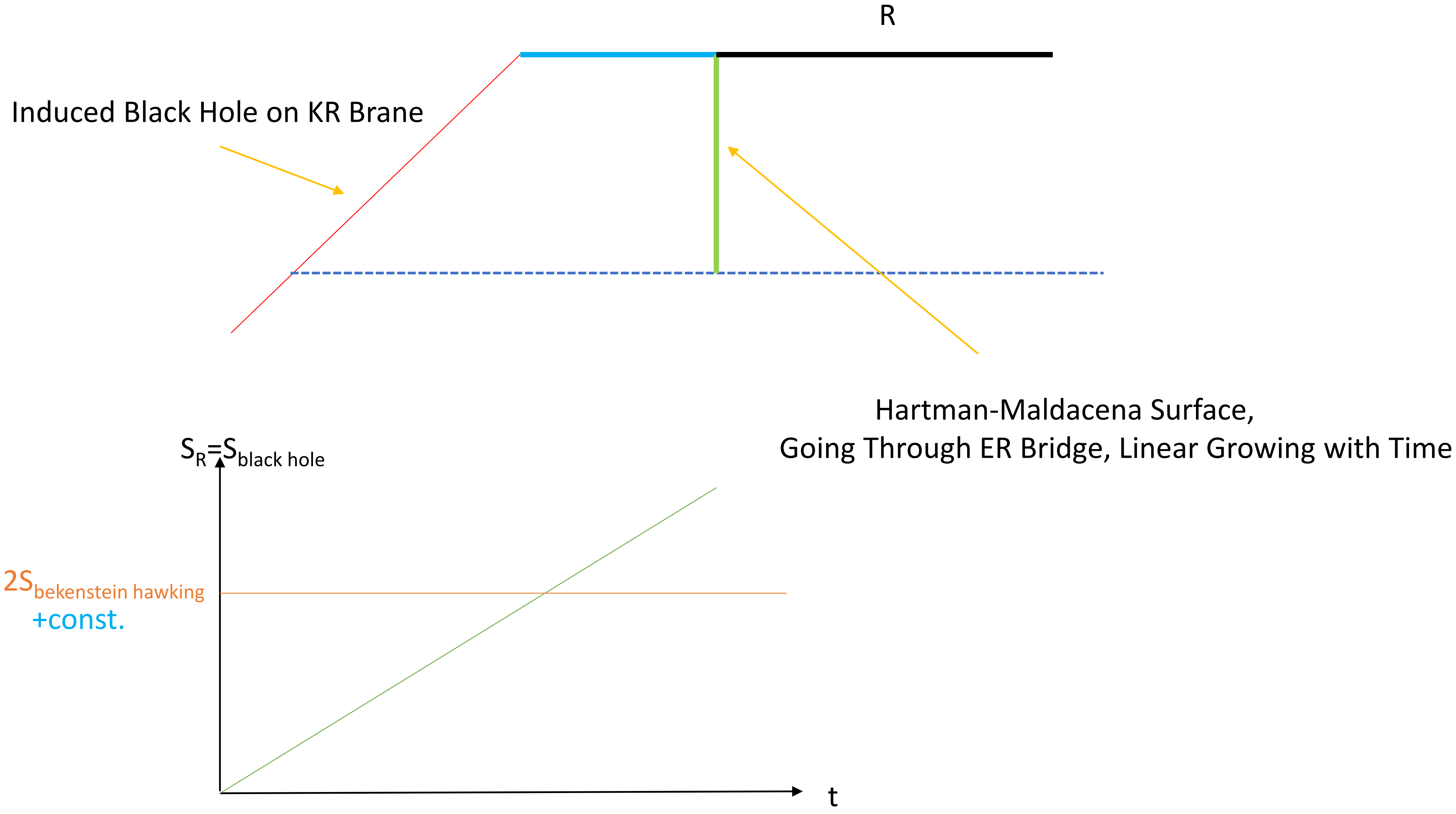
(Hartman, Maldacena 13; Mathur 14; Almheri, Mahajan and Maldacena 19)

- Eternal Black Hole in AdS:





- The Paradox:



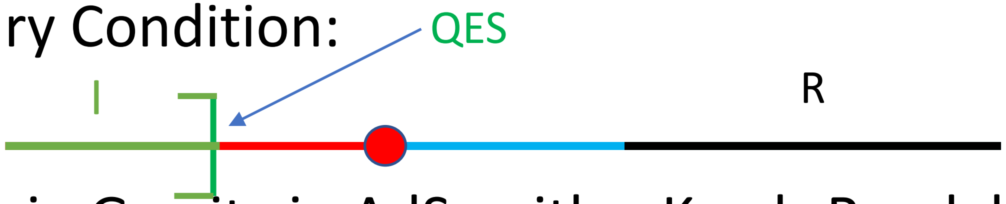
R
Induced Black Hole on KR Brane
Hartman-Maldacena Surface,
Going Through ER Bridge, Linear Growing with Time
S_R=$S_{\text{black hole}}$
$2S_{\text{bekenstein hawking}}$
+const.
t

Quantum Extremal Surface and Double Holography

(3) 4d Boundary (holographic) Conformal Field Theory (BCFT):

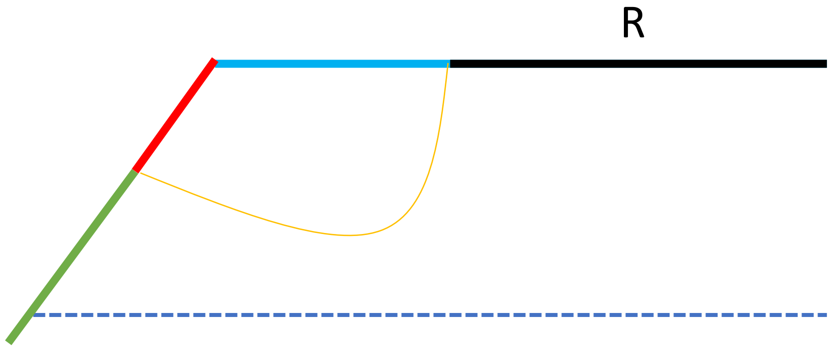
$$S_R=-\mathrm{Tr}\,\rho_R\log\rho_R$$

(2) 4d (holographic) CFT Coupled to a 4d Gravitating (holographic) CFT via Transparent Boundary Condition:



$$S_R= S_{gen}(EW(R)) = S_{gen}(R \cup I)$$

(1) 5d Einstein Gravity in AdS_5 with a Karch-Randall Brane:



$$S_R=S_{gen}(R \cup I)=S_{gen}(EW(R \cup I))=\frac{A(\partial(EW(R \cup I)))}{4G_5}$$

Analytic Resolution of the Paradox in KR

(HG and Karch 20)

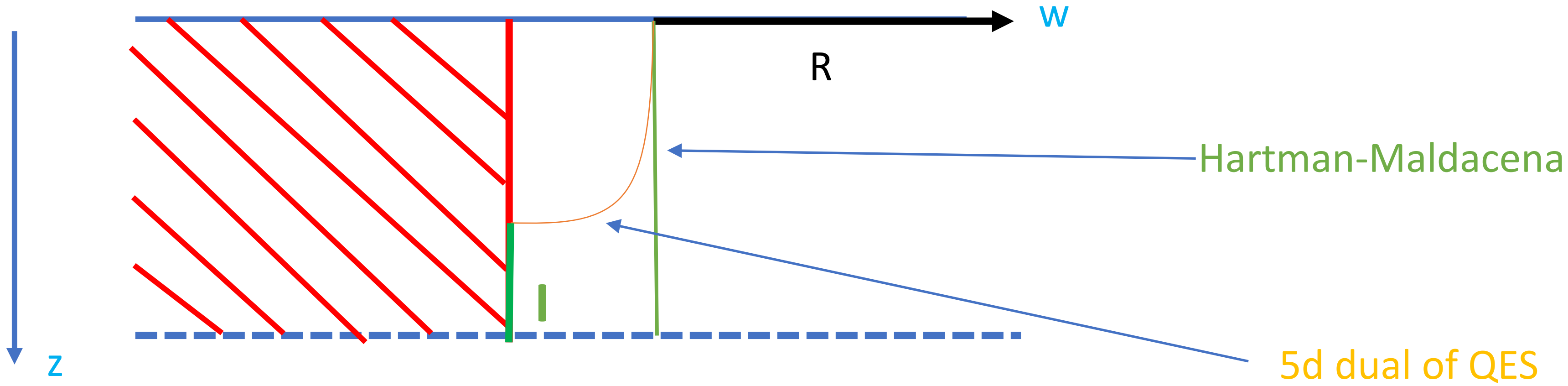
- Problem (Almheri, Mahajan and Santos 19):

  The bulk geometry in general is very complicated and requires numerical GR

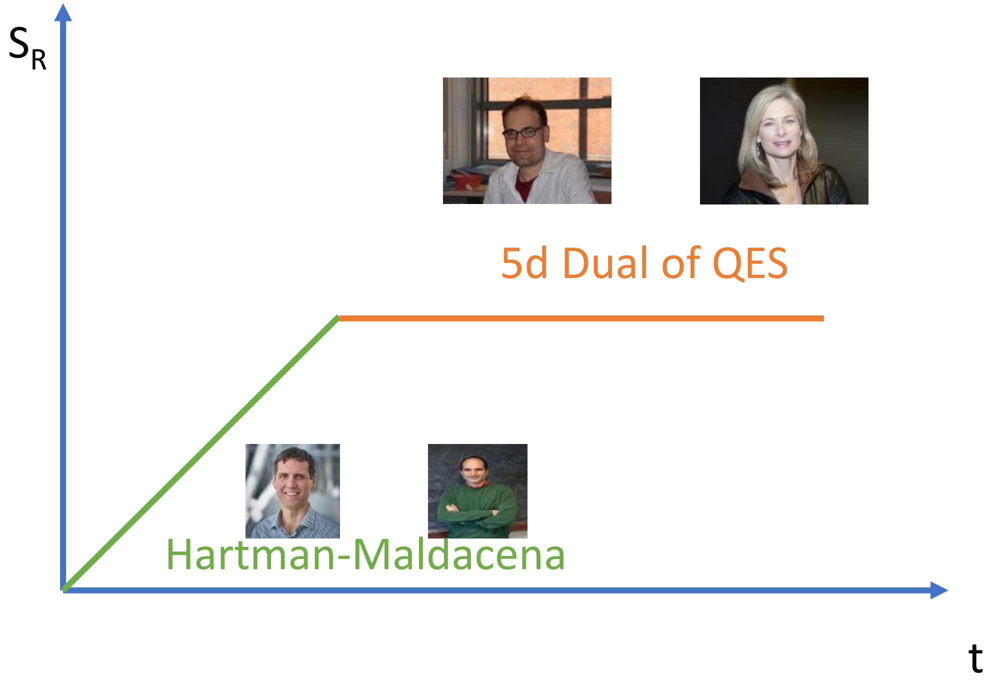
- Analytic Model (HG and Karch 20):

  Simple Observation: If the brane is tensionless then there is no back reaction at all!

$$AdS_5\,Plannar\ Black\ Hole:\ ds^2 = \frac{1}{z^2}\left(-h(z)dt^2 + \frac{dz^2}{h(z)} + dw^2 + dx^2 + dy^2\right),\ \mathrm{h(z)} = \frac{1}{1-\frac{z^4}{z_H{}^4}}$$



The Page Curve:



Massive Graviton and Islands

(HG and Karch 20)

- Massive Graviton in KR:

(2) 4d (holographic) CFT Coupled to a 4d Gravitating (holographic) CFT via a Transparent Boundary Condition.

$$\partial_\alpha T^{\alpha\beta} \neq 0$$

one-loop correction of graviton is not protected from being zero and this gives nonzero mass correction
(Porrati 02)

(1) 5d Einstein Gravity in AdS_5 with a Karch-Randall Brane:

The massless KK mode of 5d graviton is not normalizable because part of the conformal boundary is not cutoff.

- General Lessons:

a) As long as a bulk gravitational theory is coupled to a boundary bath the gravitational theory is modified at one-loop level.

b) Existing higher dimensional calculations of the Page curve of black hole radiation using AdS/CFT are all in the context of massive gravity.

(massive gravity $\longrightarrow$ Page curve)

C) Comment on massless gravity:

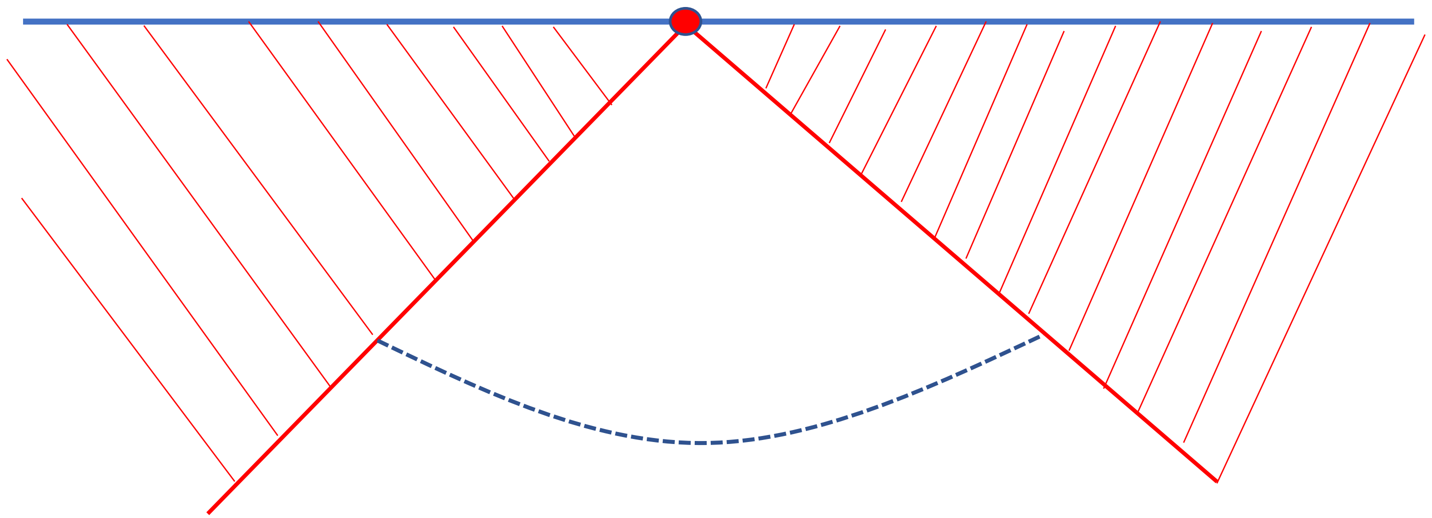
The massless mode can be recovered by cutting off the leftover part of the conformal boundary by another KR brane. But this makes the bath gravitating. And we see a constant Page curve for radiation in this context. (HG, Karch, Perez, Raju, Randall, Riojas and Shashi 20)

(massless gravity $\not\longrightarrow$ Page curve) (at least in AdS/CFT)

$\boxed{?}$

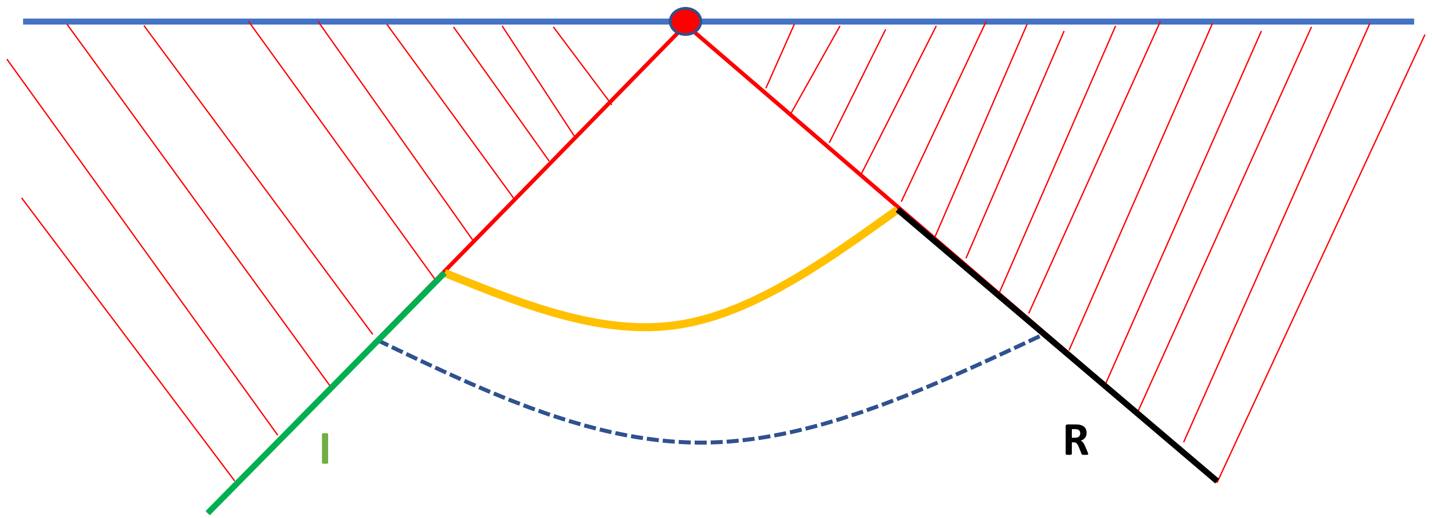
An Attempt to Massless: Information Transfer with a Gravitating Bath

(HG, Karch, Perez, Raju, Randall, Riojas and Shashi 20)



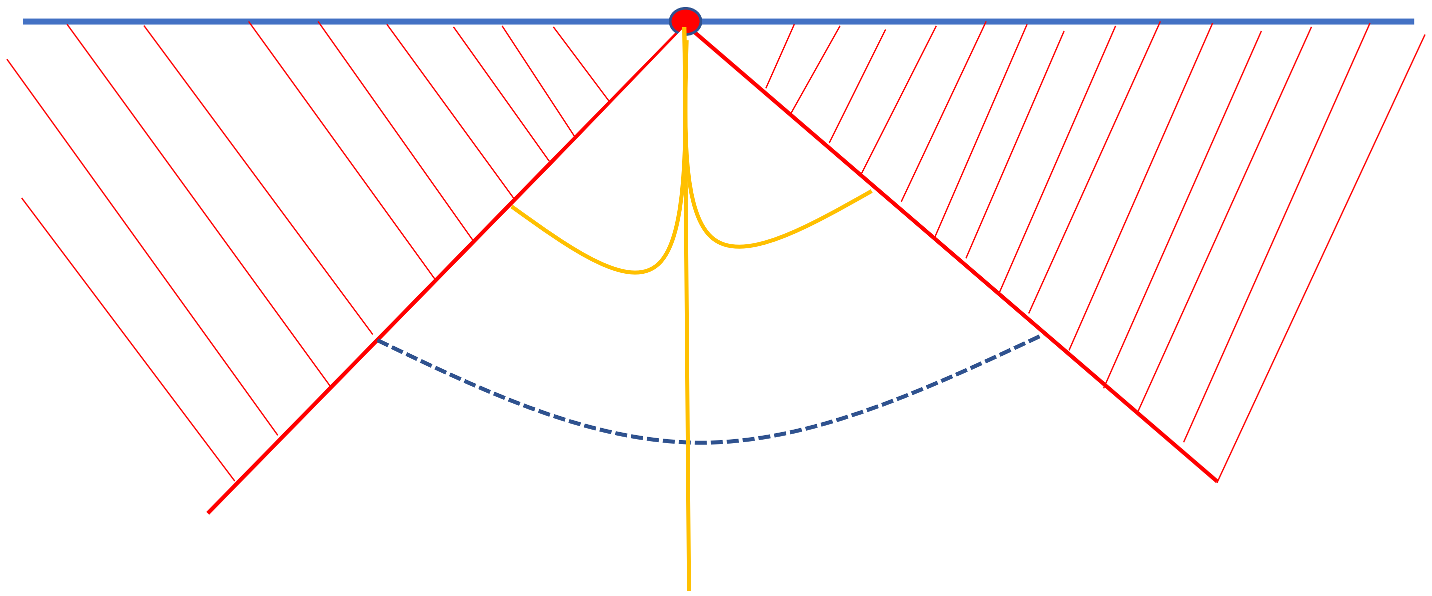
- The bulk geometry is AdS_5 black string which is foliated by AdS_4 black holes.
- We embed two Karch-Randall branes into the bulk which rescues the massless graviton mode.
- Bath is gravitating so there is no freedom to specify the radiation at the fine-grained level.

- A Dynamical Principle and a Page Curve



- $S_R = S_{gen}(R \cup I) = S_{gen}(EW(R \cup I)) = \frac{A(\partial(EW(R \cup I)))}{4G_5}$ and now we show minimize over both ∂R and ∂I.
- The RT surface is black string horizon. Therefore, we have a constant Page curve. Consistent with **Maldacena 01** and **Laddha, Prabhu, Raju and Shrivastava 20.**

• Another Question



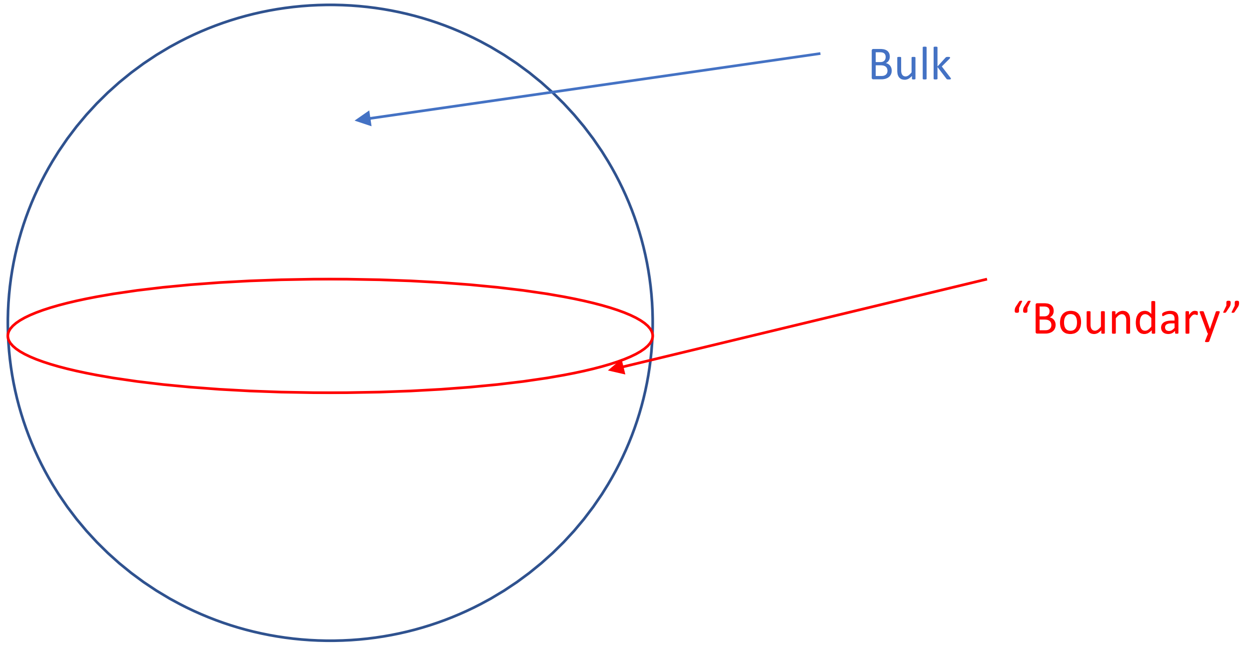
- An internal bipartition of the Hilbert space into L and R sectors.
- Three candidates entangling surfaces for $S_{L/R}$.
- Indeed, we can have a time dependent Page curve if the gravity on the brane is weak enough.
- We have an explicit CFT computation. **(Geng, Lüst, Mishra and Wakeham to appear)**

An Information Paradox in de Sitter and Its Resolution

(HG, Nomura and Sun to appear)

- The DS/dS Correspondence:

Quantum gravity in a (d+1)-dimensional de Sitter spacetime is dual to a field theory system living on a d-dimensional de Sitter space. The field theory system consists of two UV-cutoff conformal field theories coupled to dynamical gravity.



• An Information Paradox

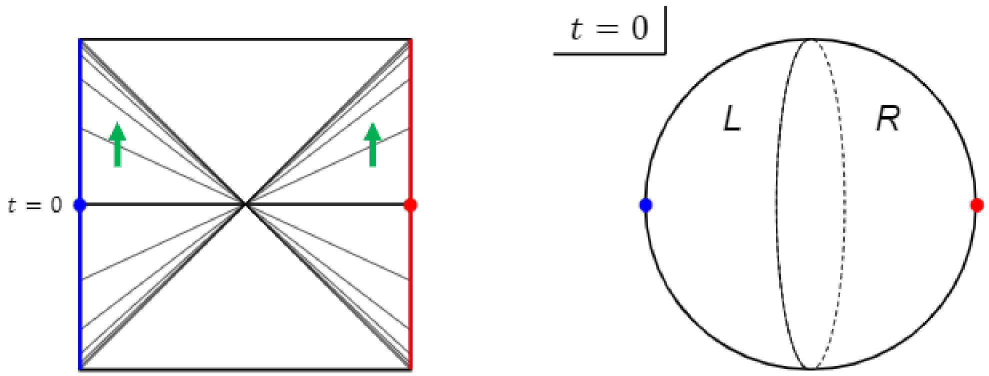


FIG. 1. *The Penrose diagram of dS spacetime in the extended static patch (left) and its spatial section at $t = 0$ (right). The blue and red dots represent the north and south poles at $t = 0$. The green arrows represent the direction of time evolution.*

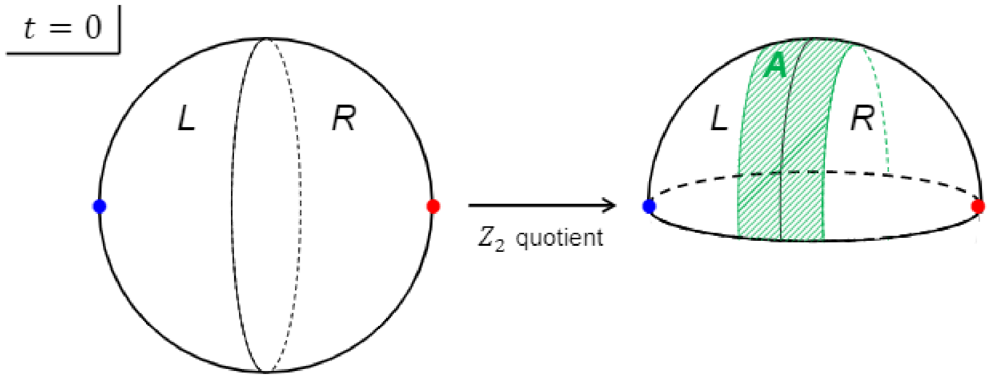


FIG. 2. *The $\mathbb{Z}_2$ orbifolding removes a half of dS_d^2. The region A, of which we calculate the entanglement entropy, at $t = 0$ is depicted as the green shaded region on the right panel.*

There would be a paradox if S_A is bigger than the minimal Hilbert space dimension of A and $\bar{A}$ or more severely if it is imaginary.

- # Embedding the Paradox into DS/dS

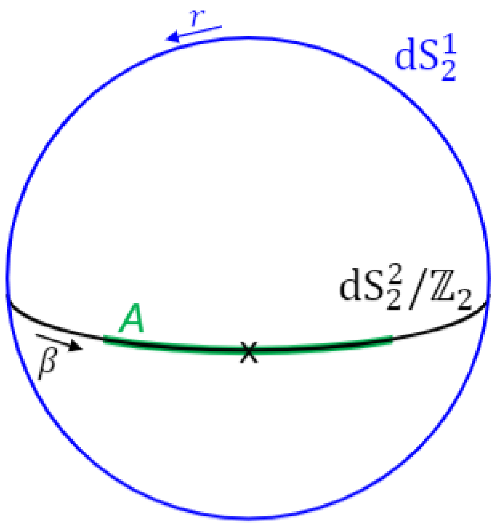
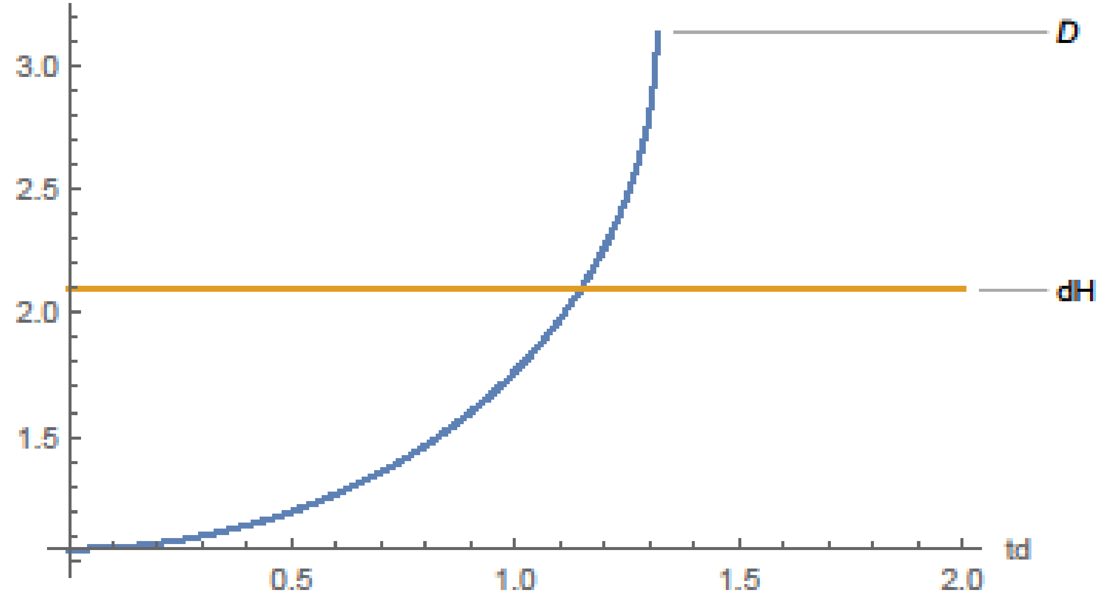


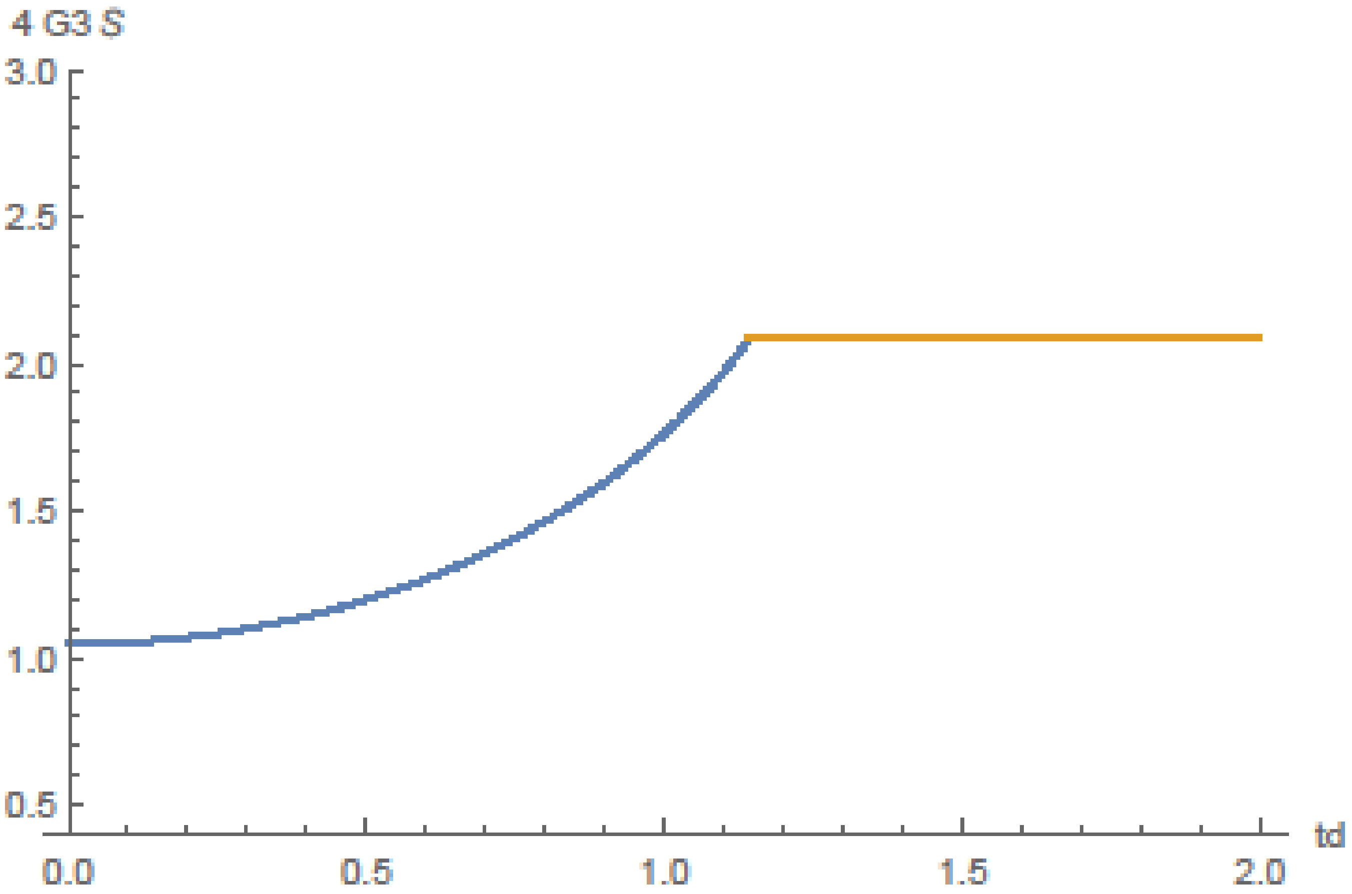
FIG. 3. *The spatial section at $t = 0$ for $d = 2$. The two intersecting dS systems are depicted as the blue circle (dS_2^1) and black semicircle ($dS_2^2/\mathbb{Z}_2$). The green segment represents region A, of which we calculate the entanglement entropy, and the cross represents the horizon at $\beta = \frac{\pi}{2}$. The holographic bulk space ($dS_3/\mathbb{Z}_2$) is the hemisphere bounded by dS_2^1.*

$$S_A = \frac{D}{4G3} = \frac{Arccos(1-2cos2(\boldsymbol{\beta}*)cosh2(t))}{4G3}$$



- Resolution

The island Ryu-Takayanagi surface is just $\bar{A}$.



- ## Lessons

1) A time-dependent Page curve.

2) The graviton is indeed massless.

3) The non-factorizability issue in Laddha, Prabhu, Raju and Shrivastava 20 is automatically avoided.

- ## A Question

Is this a fine-grained entropy?

Objection: No, it isn’t. Because there is no diffeomorphic invariant way to specify a region in a gravitating spacetime.

Support: Well, the static patch is associated to a specific observer. Hence the diffeomorphic invariance has been broken on the moment we are focusing on a static patch. The observer can always specify a region covariantly using light.

Summary

- A time-dependent Page curve of black hole radiations in AdS requires the gravitational theory to be massive. (HG, Karch 20)
- An attempt to recuse the massless graviton gives wedge holography. No time-dependent Page curve of black hole radiations. But a time-dependent Page curve for an internal bipartition of the wedge holographic dual. (HG, Karch, Perez, Raju, Randall, Riojas and Shashi 20)
- A CFT computation of the Page curve for the internal bipartition exists.

(HG, Lüst, Mishra and Wakeham to appear)

- Extension of the entanglement island program into de Sitter space.

(HG, Nomura and Sun to appear)

Thank You!